

## Grundlegung einer 2-dimensionalen semiotischen Kategorietheorie

1. Zur semiotischen Kategorietheorie vgl. Bense (1981, S. 124 ff.) und Toth (1997, S. 21 ff.). Im folgenden werden 2-dimensionalen semiotische Kategorien – und damit natürlich nur die allererlementarsten Grundlagen einer entsprechenden Kategorietheorie – für die in Toth (2015) definierten semiotischen Graphen definiert.

### 2. Semiotische Kategorien

#### 2.1. Adjazente Zählweise

0      $\rightarrow$      1                      2                       $\emptyset$

$\uparrow$     $\swarrow$                        $\otimes$     $\downarrow$     $\nwarrow$

2                       $\emptyset$                       0      $\rightarrow$      1

1      $\leftarrow$      0                       $\emptyset$                       2

$\searrow$     $\uparrow$       $\otimes$                        $\nearrow$     $\downarrow$

$\emptyset$                       2                      1      $\leftarrow$      0

0      $\rightarrow$      1                       $\emptyset$                       2

$\nwarrow$     $\downarrow$       $\otimes$                        $\swarrow$     $\uparrow$

$\emptyset$                       2                      0      $\rightarrow$      1

1      $\leftarrow$      0                      2                       $\emptyset$

$\downarrow$     $\nearrow$                        $\otimes$     $\uparrow$     $\searrow$

2                       $\emptyset$                       1      $\leftarrow$      0

## 2.2. Subjazente Zählweise

0   ←   2                      2   →   0

↓   ↗                      ⊗                      ↖   ↓

1                      ∅                      ∅                      1

1   →   2                      2   ←   1

↑   ↙                      ⊗                      ↘   ↑

0                      ∅                      ∅                      0

0                      ∅                      ∅                      0

↓   ↖                      ⊗                      ↗   ↓

1   →   2                      2   ←   1

1                      ∅                      ∅                      1

↑   ↘                      ⊗                      ↙   ↑

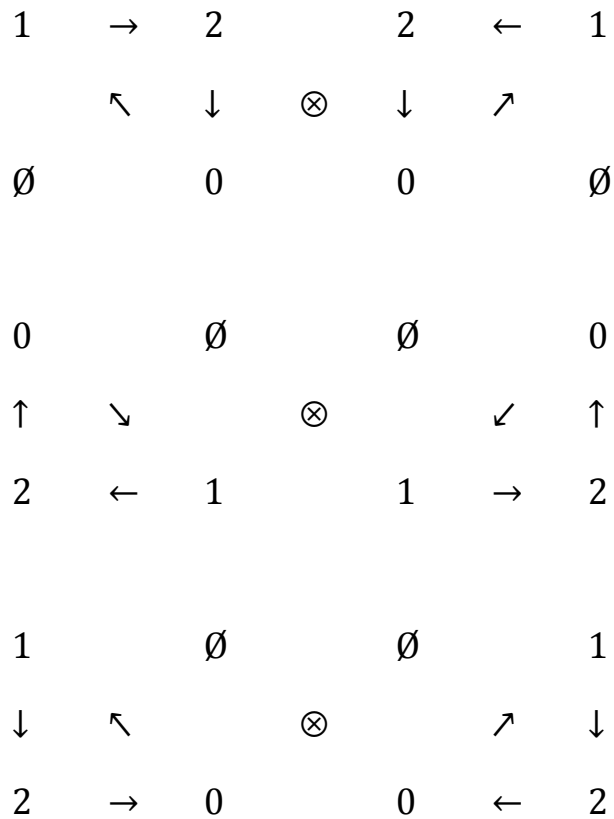
0   ←   2                      2   →   0

## 2.3. Transjazente Zählweise

0   ←   2                      2   →   0

                    ↘   ↑                      ⊗                      ↑   ↙

∅                      1                      1                      ∅



### 3. Semiotische Morphismen

Die bereits in Toth (1997) definierten semiotischen Morphismen

$$\alpha := (0 \rightarrow 1)$$

$$\beta := (1 \rightarrow 2),$$

den dazu konversen

$$\alpha^\circ = (1 \rightarrow 0)$$

$$\beta^\circ = (2 \rightarrow 1),$$

den komponierten

$$\beta\alpha = (0 \rightarrow 2)$$

$$\alpha^\circ\beta^\circ = (0 \rightarrow 2)$$

und natürlich den drei identitiven

$$\text{id}_0 := (0 \rightarrow 0)$$

$$\text{id}_1 := (1 \rightarrow 1)$$

$$\text{id}_2 := (2 \rightarrow 2)$$

sind somit in dimensionaler Abhängigkeit von den folgenden vier Paaren von Gerichtetheit zu definieren

$$(\rightarrow, \leftarrow), (\uparrow, \downarrow), (\nearrow, \swarrow) (\nwarrow, \searrow),$$

d.h. wir erhalten folgendes 8-dimensionales System semiotischer Morphismen

|               | $\alpha$             | $\alpha^\circ$              | $\beta$             | $\beta^\circ$              | $\beta\alpha$             | $\alpha^\circ\beta^\circ$              | $\text{id}_0$             | $\text{id}_1$             | $\text{id}_2$             |
|---------------|----------------------|-----------------------------|---------------------|----------------------------|---------------------------|----------------------------------------|---------------------------|---------------------------|---------------------------|
| $\rightarrow$ | $\alpha^\rightarrow$ | $\alpha^{\circ\rightarrow}$ | $\beta^\rightarrow$ | $\beta^{\circ\rightarrow}$ | $\beta\alpha^\rightarrow$ | $\alpha^\circ\beta^{\circ\rightarrow}$ | $\text{id}_0^\rightarrow$ | $\text{id}_1^\rightarrow$ | $\text{id}_2^\rightarrow$ |
| $\leftarrow$  | $\alpha^\leftarrow$  | $\alpha^{\circ\leftarrow}$  | $\beta^\leftarrow$  | $\beta^{\circ\leftarrow}$  | $\beta\alpha^\leftarrow$  | $\alpha^\circ\beta^{\circ\leftarrow}$  | $\text{id}_0^\leftarrow$  | $\text{id}_1^\leftarrow$  | $\text{id}_2^\leftarrow$  |
| $\uparrow$    | $\alpha^\uparrow$    | $\alpha^{\circ\uparrow}$    | $\beta^\uparrow$    | $\beta^{\circ\uparrow}$    | $\beta\alpha^\uparrow$    | $\alpha^\circ\beta^{\circ\uparrow}$    | $\text{id}_0^\uparrow$    | $\text{id}_1^\uparrow$    | $\text{id}_2^\uparrow$    |
| $\downarrow$  | $\alpha^\downarrow$  | $\alpha^{\circ\downarrow}$  | $\beta^\downarrow$  | $\beta^{\circ\downarrow}$  | $\beta\alpha^\downarrow$  | $\alpha^\circ\beta^{\circ\downarrow}$  | $\text{id}_0^\downarrow$  | $\text{id}_1^\downarrow$  | $\text{id}_2^\downarrow$  |
| $\nearrow$    | $\alpha^\nearrow$    | $\alpha^{\circ\nearrow}$    | $\beta^\nearrow$    | $\beta^{\circ\nearrow}$    | $\beta\alpha^\nearrow$    | $\alpha^\circ\beta^{\circ\nearrow}$    | $\text{id}_0^\nearrow$    | $\text{id}_1^\nearrow$    | $\text{id}_2^\nearrow$    |
| $\swarrow$    | $\alpha^\swarrow$    | $\alpha^{\circ\swarrow}$    | $\beta^\swarrow$    | $\beta^{\circ\swarrow}$    | $\beta\alpha^\swarrow$    | $\alpha^\circ\beta^{\circ\swarrow}$    | $\text{id}_0^\swarrow$    | $\text{id}_1^\swarrow$    | $\text{id}_2^\swarrow$    |
| $\nwarrow$    | $\alpha^\nwarrow$    | $\alpha^{\circ\nwarrow}$    | $\beta^\nwarrow$    | $\beta^{\circ\nwarrow}$    | $\beta\alpha^\nwarrow$    | $\alpha^\circ\beta^{\circ\nwarrow}$    | $\text{id}_0^\nwarrow$    | $\text{id}_1^\nwarrow$    | $\text{id}_2^\nwarrow$    |
| $\searrow$    | $\alpha^\searrow$    | $\alpha^{\circ\searrow}$    | $\beta^\searrow$    | $\beta^{\circ\searrow}$    | $\beta\alpha^\searrow$    | $\alpha^\circ\beta^{\circ\searrow}$    | $\text{id}_0^\searrow$    | $\text{id}_1^\searrow$    | $\text{id}_2^\searrow$    |

für die drei in 2 Dimensionen unterscheidbaren Zählweisen.

## Literatur

Bense, Max, Axiomatik und Semiotik. Baden-Baden 1981

Toth, Alfred, Entwurf einer Semiotisch-Relationalen Grammatik. Tübingen 1997

Toth, Alfred, Semiotische Graphen in 2-dimensionalen Raumfeldern. In: Electronic Journal for Mathematical Semiotics, 2015 20.5.2015